NAG Toolbox for MATLAB

g07ca

1 Purpose

g07ca computes a t-test statistic to test for a difference in means between two Normal populations, together with a confidence interval for the difference between the means.

2 Syntax

```
[t, df, prob, dl, du, ifail] = g07ca(tail, equal, nx, ny, xmean, ymean,
xstd, ystd, clevel)
```

3 Description

Consider two independent samples, denoted by X and Y, of size n_x and n_y drawn from two Normal populations with means μ_x and μ_y, and variances σ_x^2 and σ_y^2 respectively. Denote the sample means by $\bar{x}$ and $\bar{y}$ and the sample variances by s_x^2 and s_y^2 respectively.

g07ca calculates a test statistic and its significance level to test the null hypothesis $H_0 : \mu_x = \mu_y$, together with upper and lower confidence limits for $\mu_x - \mu_y$. The test used depends on whether or not the two population variances are assumed to be equal.

1. It is assumed that the two variances are equal, that is $\sigma_x^2 = \sigma_y^2$.

   The test used is the two sample t-test. The test statistic t is defined by;

$$t_{\text{obs}} = \frac{\bar{x} - \bar{y}}{s\sqrt{(1/n_x) + \left(1/n_y\right)}}$$

   where

$$s^2 = \frac{(n_x - 1)s_x^2 + \left(n_y - 1\right)s_y^2}{n_x + n_y - 2}$$

   is the pooled variance of the two samples.

   Under the null hypothesis H_0 this test statistic has a t-distribution with $\left(n_x + n_y - 2\right)$ degrees of freedom.

   The test of H_0 is carried out against one of three possible alternatives;

   $H_1 : \mu_x \neq \mu_y$; the significance level, $p = P(t \geq |t_{\text{obs}}|)$, i.e., a two tailed probability.

   $H_1 : \mu_x > \mu_y$; the significance level, $p = P(t \geq t_{\text{obs}})$, i.e., an upper tail probability.

   $H_1 : \mu_x < \mu_y$; the significance level, $p = P(t \leq t_{\text{obs}})$, i.e., a lower tail probability.

   Upper and lower $100(1 - \alpha)\%$ confidence limits for $\mu_x - \mu_y$ are calculated as:

$$(\bar{x} - \bar{y}) \pm t_{1-\alpha/2}s\sqrt{(1/n_x) + \left(1/n_y\right)}.$$

   where $t_{1-\alpha/2}$ is the $100(1 - \alpha/2)$ percentage point of the t-distribution with $(n_x + n_y - 2)$ degrees of freedom.

2. It is not assumed that the two variances are equal.

   If the population variances are not equal the usual two sample t-statistic no longer has a t-distribution and an approximate test is used.

This problem is often referred to as the Behrens–Fisher problem, see Kendall and Stuart 1969. The test used here is based on Satterthwaites procedure. To test the null hypothesis the test statistic t' is used where

$$t'_{\mathrm{obs}} = \frac{\bar{x} - \bar{y}}{\mathrm{se}(\bar{x} - \bar{y})}$$

where $\mathrm{se}(\bar{x} - \bar{y}) = \sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}$.

A t-distribution with f degrees of freedom is used to approximate the distribution of t' where

$$f = \frac{\mathrm{se}\,(\bar{x} - \bar{y})^4}{\frac{\left(s_x^2/n_x\right)^2}{(n_x - 1)} + \frac{\left(s_y^2/n_y\right)^2}{\left(n_y - 1\right)}}.$$

The test of H_0 is carried out against one of the three alternative hypotheses described above, replacing t by t' and t_{obs} by t'_{obs}.

Upper and lower $100(1 - \alpha)\%$ confidence limits for $\mu_x - \mu_y$ are calculated as:

$$(\bar{x} - \bar{y}) \pm t_{1-\alpha/2}\,\mathrm{se}(x - \bar{y}).$$

where $t_{1-\alpha/2}$ is the $100(1 - \alpha/2)$ percentage point of the t-distribution with f degrees of freedom.

4 References

Johnson M G and Kotz A 1969 *The Encyclopedia of Statistics* **2** Griffin

Kendall M G and Stuart A 1969 *The Advanced Theory of Statistics (Volume 1)* (3rd Edition) Griffin

Snedecor G W and Cochran W G 1967 *Statistical Methods* Iowa State University Press

5 Parameters

5.1 Compulsory Input Parameters

1: **tail – string**

Indicates which tail probability is to be calculated, and thus which alternative hypothesis is to be used.

tail = 'T'

The two tail probability, i.e., $H_1 : \mu_x \neq \mu_y$.

tail = 'U'

The upper tail probability, i.e., $H_1 : \mu_x > \mu_y$.

tail = 'L'

The lower tail probability, i.e., $H_1 : \mu_x < \mu_y$.

Constraint: **tail** = 'T', 'U' or 'L'.

2: **equal – string**

Indicates whether the population variances are assumed to be equal or not.

equal = 'E'

The population variances are assumed to be equal, that is $\sigma_x^2 = \sigma_y^2$.

equal = 'U'

The population variances are not assumed to be equal.

Constraint: **equal** = 'E' or 'U'.

3: **nx – int32 scalar**

n_x, the size of the X sample.

Constraint: **nx** ≥ 2.

4: **ny – int32 scalar**

n_y, the size of the Y sample.

Constraint: **ny** ≥ 2.

5: **xmean – double scalar**

$\bar{x}$, the mean of the X sample.

6: **ymean – double scalar**

$\bar{y}$, the mean of the Y sample.

7: **xstd – double scalar**

s_x, the standard deviation of the X sample.

Constraint: **xstd** > 0.0.

8: **ystd – double scalar**

s_y, the standard deviation of the Y sample.

Constraint: **ystd** > 0.0.

9: **clevel – double scalar**

The confidence level, $1 - \alpha$, for the specified tail. For example **clevel** $= 0.95$ will give a 95% confidence interval.

Constraint: $0.0 <$ **clevel** < 1.0.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **t – double scalar**

Contains the test statistic, t_{obs} or t'_{obs}.

2: **df – double scalar**

Contains the degrees of freedom for the test statistic.

3: **prob – double scalar**

Contains the significance level, that is the tail probability, p, as defined by **tail**.

4: **dl – double scalar**

Contains the lower confidence limit for $\mu_x - \mu_y$.

5: **du – double scalar**

Contains the upper confidence limit for $\mu_x - \mu_y$.

6: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **tail** $\neq$ 'T', 'U' or 'L',
or **equal** $\neq$ 'E' or 'U',
or **nx** < 2,
or **ny** < 2,
or **xstd** ≤ 0.0,
or **ystd** ≤ 0.0,
or **clevel** ≤ 0.0,
or **clevel** ≥ 1.0.

7 Accuracy

The computed probability and the confidence limits should be accurate to approximately five significant figures.

8 Further Comments

The sample means and standard deviations can be computed using g01aa.

9 Example

```
tail = 'Two';
equal = 'Equal';
nx = int32(4);
ny = int32(8);
xmean = 25;
ymean = 21;
xstd = 0.8185;
ystd = 4.2083;
clevel = 0.95;
[t, df, prob, dl, du, ifail] = ...
    g07ca(tail, equal, nx, ny, xmean, ymean, xstd, ystd, clevel)
```

```
t =
    1.8403
df =
    10
prob =
    0.0955
```

```
dl =
   -0.8429
du =
    8.8429
ifail =
           0
```